THE KING'S SCHOOL

2009
Higher School Certificate
Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

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Total marks – 84
Attempt Questions 1-7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the table of standard integrals to find $\int \frac{1}{\sqrt{x^2 - 12}}\,dx$ **1**

(b) Find the acute angle between the lines with gradients 9 and $\frac{4}{5}$. **2**

(c) (i) Show that $\frac{1}{1 + e^{-x}} = \frac{e^x}{1 + e^x}$ **1**

(ii) Hence, or otherwise, evaluate $\int_0^1 \frac{1}{1 + e^{-x}}\,dx$ **2**

(d) Find the remainder $R(x)$ when $P(x) = x^3 + x^2 + x$ is divided by $x^2 - 12$ **2**

(e) (i) Find the domain of the function $y = \ln(2x - 1) - \ln(x + 1)$ **2**

(ii) Hence, or otherwise, solve the inequality $\frac{2x - 1}{x + 1} > 0$ **2**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Use the substitution $x = \sin\theta$ to show that

$$J = \int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1 - x^2}}\, dx = \int_0^{\frac{\pi}{6}} \sin^2\theta\, d\theta$$ **3**

(ii) Hence, or otherwise, evaluate *J*. **3**

(b) The cubic equation $9x^3 + Ax + 2 = 0$, where *A* is real, has two roots whose sum is 1 and a third root α.

(i) Find the value of *A*. **2**

(ii) Find the product of the two roots whose sum is 1. **1**

(c) (i) Write $\sin 2\theta$ in terms of $t = \tan\theta$ **1**

(ii) Prove that $\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot\theta$ **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int \frac{dx}{9 + 4x^2}$ **2**

(b) The root of $f(x) = 0.4x - e^{-x^2} = 0$ is near $x = 1$

Use Newton's Method once to find a 2 decimal place approximation to this root. **3**

(c) A particle moves on the *x* axis so that at any time *t* its velocity is *v*.

Prove that the acceleration $\frac{dv}{dt} = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ **2**

(d) A particle is oscillating about $x = 0$ in simple harmonic motion.

Its acceleration $\ddot{x} = -n^2x, \ n > 0,$ and initially it is at rest at $x = A > 0$.

(i) By using integration, prove that $v^2 = n^2(A^2 - x^2)$, where *v* is its velocity. **2**

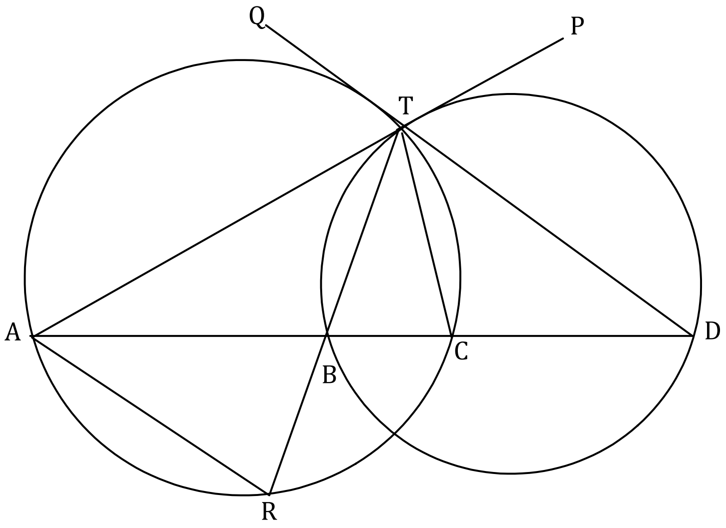
(ii) When $x = 6, \ v = -2$ and when $x = 3, \ v = -4$.

Find the amplitude and period of the motion. **3**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



ATP is a tangent to the circle TBD.
DTQ is a tangent to the circle TAC.

TB produced meets the circle TAC at R.

(i) Explain why $\angle \mathrm{PTD} = \angle \mathrm{TBD}$. **1**

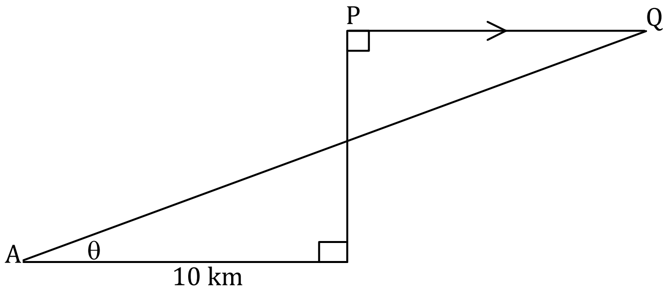
(ii) Explain why $\angle \mathrm{PTD} = \angle \mathrm{QTA}$. **1**

(iii) Deduce that TB = TC. **2**

(iv) Prove that $\Delta \mathrm{ABR}$ isosceles. **2**

Question 4 continues on next page

(b)



A plane P is flying due East at a constant height of 8 km and a constant speed of 6km/min.

The plane is being tracked from a point A on the ground which was initially 10 km due West of the plane P.

Let Q be the position of the plane after *t* minutes and let the angle of elevation from A be θ at this time.

(i) Show that $\tan\theta = \dfrac{4}{5 + 3t}$ **2**

(ii) Deduce that $\dfrac{d\theta}{dt} = \dfrac{-12}{\sec^2\theta(5 + 3t)^2}$ **2**

(iii) Find the rate at which the angle of elevation from A is decreasing after 1 minute. Give your answer in degrees/minute. **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

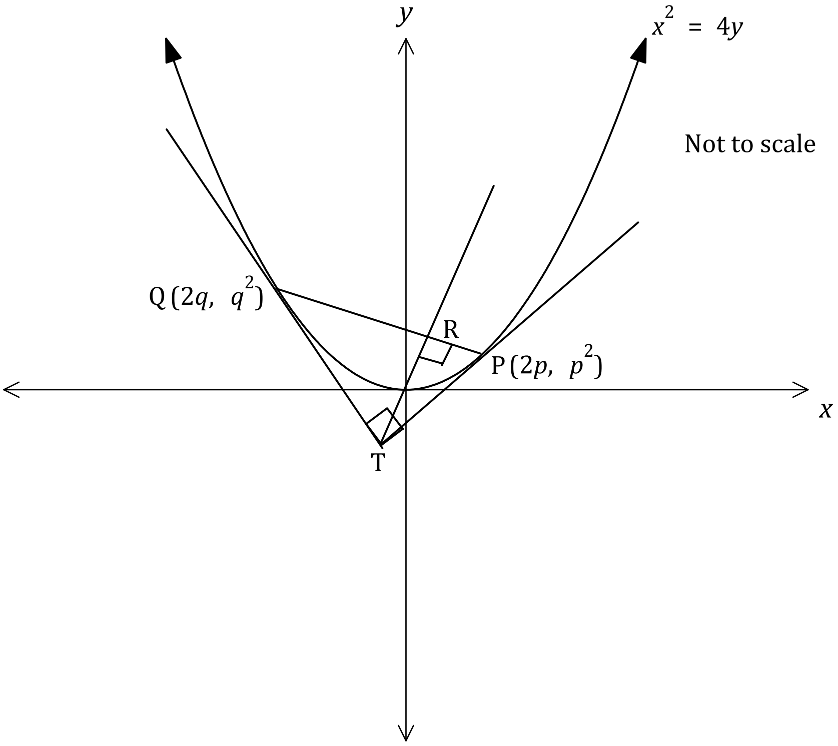
(a) (i) Show that $\dfrac{\dbinom{20}{k-1}}{\dbinom{20}{k}} = \dfrac{k}{21-k}$ **1**

(ii) In the binomial expansion of $\left(x^2 + \dfrac{b}{x}\right)^{20}$ the coefficients of x^7 and x^4 are equal.

Find the value of b. **3**

Question 5 continues on the next page

(b)



The tangents at P $(2p,\ p^2)$ and Q $(2q,\ q^2)$ on the parabola $x^2 = 4y$ meet at right angles at T.

The equation of the tangent at P is $y = px - p^2$.
The equation of the chord PQ is $y = \frac{p+q}{2}x - pq$.

[DO NOT PROVE THESE]

(i) Write down the gradient of the tangent at P. **1**

(ii) Hence show that $pq = -1$. **1**

(iii) Prove that the tangents at P and Q meet at T $(p + q,\ -1)$. **2**

(iv) Show that PQ is a focal chord. **1**

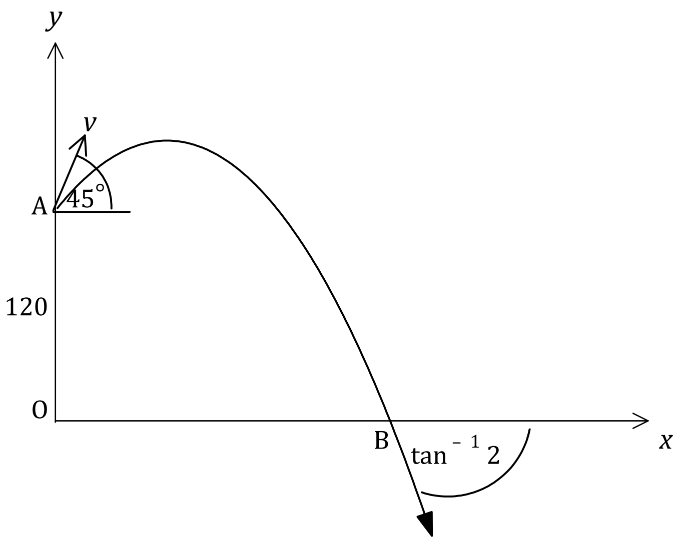
(v) A line is drawn from T to meet the chord PQ at right angles at R.
Prove that the equation of TR is $y = \frac{-2}{p+q}x + 1$. **2**

(vi) Find the coordinates of R. **1**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



A projectile is fired from a point A, 120 m above horizontal ground, with a speed of v m/s and elevation 45°. It lands on the horizontal ground at B making an angle of $\tan^{-1} 2$ with the horizontal.

The equations of motion of the projectile are:

$$\ddot{x} = 0 \qquad \text{and} \qquad \ddot{y} = -10$$

$$\dot{x} = \frac{v}{\sqrt{2}} \qquad \qquad \dot{y} = -10t + \frac{v}{\sqrt{2}}$$

$$x = \frac{v}{\sqrt{2}}t \qquad \qquad y = -5t^2 + \frac{v}{\sqrt{2}}t + 120$$

[DO NOT PROVE THESE]

Suppose the projectile lands at B at time $t = T$.

(i) State the vertical component of velocity at B. **1**

(ii) Prove that $10\sqrt{2}\ T = 3v$. **2**

(iii) Find v. **2**

(iv) Find the greatest height above horizontal ground that the particle reaches. **2**

Question 6 continues on the next page

(b) Let n be any integer ≥ 2 and x be any integer $\neq 0$

Let $E(n) = (1 + x)^n - nx - 1$

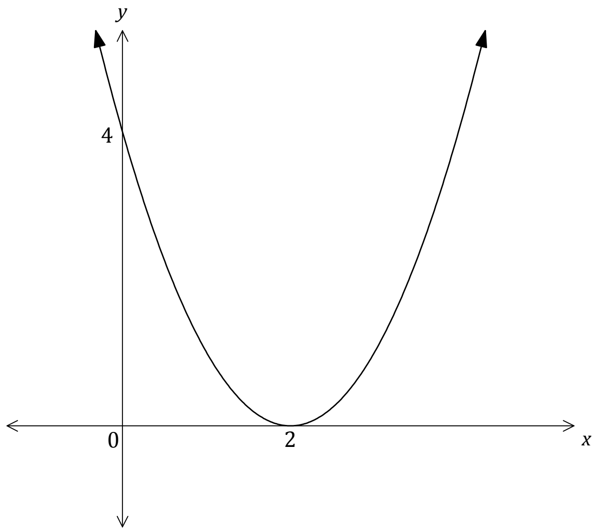
(i) Show that $E(n)$ is divisible by x^2 by using the binomial expansion. **1**

(ii) Prove that $E(n)$ is divisible by x^2 by using mathematical induction, $n \geq 2$ **4**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



The sketch shows the parabola $y = f(x) = (x - 2)^2$

(i) Explain why for $x \geq 2$ that an inverse function, $y = f^{-1}(x)$, exists. **1**

(ii) State the domain and range of this inverse function. **1**

(iii) At what point will $y = f(x)$ meet $y = f^{-1}(x)$? **1**

(iv) If $k < 2$, find, in simplest form, $f^{-1}(f(k))$ **2**

(b) Let $f(x) = 2\cos^{-1} x$, $-1 \leq x \leq 1$

and $g(x) = \sin^{-1}(2x^2 - 1)$, $-1 \leq x \leq 0$

(i) Sketch the graph of $y = f(x)$ **1**

(ii) Show that $g'(x) = f'(x)$, $-1 < x < 0$ **3**

(iii) Hence, or otherwise, express $g(x)$ in terms of $f(x)$. **2**

(iv) Sketch the graph of $y = g(x)$. **1**

End of Examination Paper

Standard Integrals

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

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Question	Algebra and Number		Geometry		Functions		Trigonometry		Differential Calculus		Integral Calculus		Total
1	(c)(i), (d), (e)	7					(b)	2			(a), (c)(ii)	3	12
2	(b)	3					(c)	3			(a)	6	12
3					(d)(ii)	3			(b), (c)	5	(a), (d)(i)	4	12
4			(a)	6	(b)(iii)	2	(b)(i)	2	(b)(ii)	2			12
5	(a)	4			(b)	8							12
6	(b)	5			(a)	7							12
7					(a), (b)(i), (iii), (iv)	9			(b)(ii)	3			12
Total		19		6		29		7		10		13	84

TKS EXTENSION 1 SOLUTIONS TRIAL 2009

Question 1

(a) $\ln\left(x + \sqrt{x^2-12}\right) \quad (+c)$

(b) $\tan\alpha = \dfrac{9-\frac{4}{5}}{1+9\cdot\frac{4}{5}} = \dfrac{45-4}{5+36} = 1$

$\therefore \alpha = \dfrac{\pi}{4}$

(c) (i) $\dfrac{1}{1+e^{-x}} = \dfrac{e^x}{e^x(1+e^{-x})} = \dfrac{e^x}{e^x+1}$

(ii) $I = \displaystyle\int_0^1 \frac{e^x}{e^x+1}\,dx = \Big[\ln(e^x+1)\Big]_0^1$

$= \ln(e+1) - \ln 2$

(d)
$$\begin{array}{rl} & x \quad\ +1 \\ x^2-12 \!\!\!& \overline{)\,x^3 \quad +x^2 \quad +x} \\ & \qquad\qquad -12x \quad -12 \\ \cline{2-2} & \qquad\qquad 13x \qquad 12 \end{array}$$

$\Rightarrow Q(x) = x+1$

& $R(x) = 13x + 12$

(e) (i) $2x-1 > 0$ and $x+1 > 0$

i.e. $x > \frac{1}{2}$ and $x > -1$

$\therefore$ domain is $x > \frac{1}{2}$

(ii) From (i), $y = \ln\left(\dfrac{2x-1}{x+1}\right)$

$\Rightarrow \dfrac{2x-1}{x+1} > 0$ if $x > \frac{1}{2}$ or $x < -1$

Question 2

(a) (i) $x = \sin\theta$ $\qquad x = 0,\ \theta = 0$

$$\frac{dx}{d\theta} = \cos\theta \qquad x = \frac{1}{2},\ \theta = \frac{\pi}{6}$$

$$\therefore\ J = \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta$$

$$= \int_0^{\pi/6} \frac{\sin^2\theta\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta = \int_0^{\frac{\pi}{6}} \sin^2\theta\ d\theta$$

(ii) $J = \frac{1}{2}\int_0^{\frac{\pi}{6}} 1-\cos 2\theta\ d\theta$

$$= \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/6}$$

$$= \frac{1}{2}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} - (0)\right) \text{ will do}$$

$$= \frac{2\pi - 3\sqrt{3}}{24}$$

(b) (i) Sum of roots $= 1 + \alpha = 0 \quad \therefore\ \alpha = -1$

$\therefore\ -9 - A + 2 = 0 \Rightarrow A = -7$

(ii) $\alpha\beta\gamma = -\frac{2}{9} \Rightarrow \beta\gamma = \frac{2}{9}$

(c) (i) $\frac{2t}{1+t^2}$

(ii) Put $t = \tan\theta$

Then $\operatorname{cosec} 2\theta + \cot 2\theta = \frac{1+t^2}{2t} + \frac{1-t^2}{2t}$

$$= \frac{2}{2t} = \frac{1}{\tan\theta}$$

$$= \cot\theta$$

Question 3

(a) $$\int \frac{dx}{3^2+(2x)^2} = \frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right)\cdot\frac{1}{2} \qquad (+c)$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)$$

(b) $f'(x) = 0.4 + 2xe^{-x^2}$

$$\therefore x_1 = 1 - \frac{0.4 - e^{-1}}{0.4 + 2e^{-1}} = 0.97, \text{ 2 d.p.}$$

(c) $$\frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx}$$

$$= \frac{d(\frac{1}{2}v^2)}{dv}\cdot\frac{dv}{dx} = \frac{d(\frac{1}{2}v^2)}{dx}$$

(d) (i) $$\frac{d(\frac{1}{2}v^2)}{dx} = -n^2x$$

$$\therefore \frac{1}{2}v^2 = -n^2\frac{x^2}{2} + c$$

$$: 0 = -n^2\frac{A^2}{2} + c, \quad c = n^2\frac{A^2}{2}$$

$$\therefore v^2 = -n^2x^2 + n^2A^2 = n^2(A^2 - x^2)$$

(ii) $$\therefore 4 = n^2(A^2 - 36)$$

and $$16 = n^2(A^2 - 9)$$

$$\therefore \frac{A^2-9}{A^2-36} = 4 \Rightarrow A^2 - 9 = 4A^2 - 144$$

$$3A^2 = 135$$

$$A^2 = 45$$

$$\therefore \text{amp } A = \sqrt{45} = 3\sqrt{5}$$

$$n^2 = \frac{4}{45-36} = \frac{4}{9} \Rightarrow n = \frac{2}{3}$$

$$\therefore \text{period } T = \frac{2\pi}{2}.3 = 3\pi$$

Question 4

(a) (i) Alternate segment theorem in circle BDT

(ii) Vertically opposite angles

(iii) $\angle QTA = \angle TCA$, alt. seg thm in circle TAC

$\therefore \angle TCA = \angle TBD$ from (i) & (ii)

$\therefore \Delta TBC$ is isosceles, base angles equal

$\therefore TB = TC$

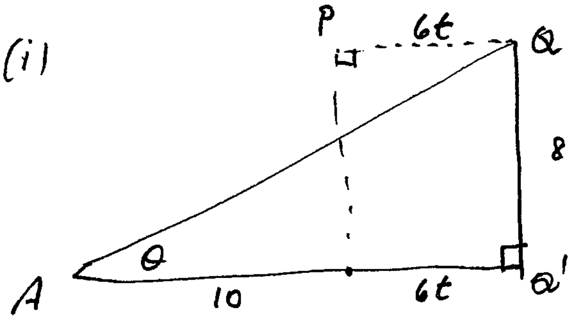
(iv) $\angle ABR = \angle TBC$, vert opp $\angle$s

$\angle ARB = \angle TCB$, $\angle$s in same segment, circle TAC

$\therefore$ From (iii), $\angle ABR = \angle ARB$

$\Rightarrow \Delta ABR$ is isosceles, base angles =

(b) (i)



$$\therefore \tan\theta = \frac{8}{10+6t} = \frac{4}{5+3t}$$

(ii) $$\frac{d \tan\theta}{dt} = \frac{d \tan\theta}{d\theta} \cdot \frac{d\theta}{dt} = \frac{d}{dt} 4(5+3t)^{-1}$$

$$\therefore \sec^2\theta \frac{d\theta}{dt} = -4(5+3t)^{-2} \cdot 3$$

i.e. $$\frac{d\theta}{dt} = \frac{-12}{\sec^2\theta (5+3t)^2}$$

(iii) $t=1$, $\tan\theta = \frac{4}{8} = \frac{1}{2}$ $\quad \therefore \frac{d\theta}{dt} = \frac{-12}{(1+\frac{1}{4})8^2}$ rad/min

$$= -\frac{3}{20} \times \frac{180}{\pi} \text{ °/min} = -\frac{27}{\pi} \text{ °/min}$$

ie decreasing at $\frac{27}{\pi}$ °/min

Question 5

(a) (i) $$\frac{\binom{20}{k-1}}{\binom{20}{k}} = \frac{20!\,(20-k)!\,k!}{(21-k)!\,(k-1)!\,20!} = \frac{k}{21-k}$$

(ii) $$u_{k+1} = \binom{20}{k}(x^2)^{20-k}\left(\frac{b}{x}\right)^k$$

$$= \binom{20}{k} x^{40-3k} b^k$$

Put $40-3k=7 \Rightarrow k=11$

$40-3k=4 \Rightarrow k=12$

$$\therefore \binom{20}{11} b^{11} = \binom{20}{12} b^{12}$$

$$\Rightarrow b = \frac{\binom{20}{11}}{\binom{20}{12}} = \frac{12}{21-12} \quad \text{from (i)}$$

$$= \frac{4}{3}$$

(b) (i) p

(ii) gradient of tangent at Q is q

$\therefore pq = -1$ since $\angle PTQ = 90°$

(iii) at T, $px - p^2 = qx - q^2$

$$\therefore (p-q)x = p^2 - q^2 = (p-q)(p+q)$$

$$\therefore x = p+q$$

$$y = p(p+q) - p^2 = pq = -1$$

$$\therefore T = (p+q, -1)$$

(iv) From data and (ii),

chord PQ is $y = \frac{p+q}{2}x + 1$

$\Rightarrow (0,1)$ is on this chord

But the focus is $(0,1)$ $\therefore$ result.

(v) grd chord PQ is $\frac{p+q}{2}$

$\therefore$ gradient of $TR = -\frac{2}{p+q}$

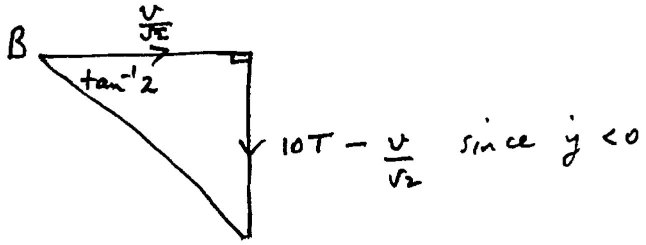
$\therefore$ TR is $y = -\frac{2}{p+q}x + 1$ since $(p+q, -1)$ is on it and satisfies this equation

(vi) Comparing chord PQ and TR, clearly $R = (0,1)$, the focus.

Question 6

(a) (i) $\dot{y} = -10T + \frac{v}{\sqrt{2}}$

(ii)



$$\Rightarrow \quad 2 = \frac{10T - \frac{v}{\sqrt{2}}}{\frac{v}{\sqrt{2}}} = \frac{10\sqrt{2}T - v}{v}$$

$$\therefore \quad 2v = 10\sqrt{2}T - v$$

$$\Rightarrow \quad 10\sqrt{2}T = 3v$$

(iii) at B, $y = 0$

$$\therefore \quad 0 = -5T^2 + \frac{v}{\sqrt{2}}T + 120$$

$$\Rightarrow \quad 0 = -5.\left(\frac{3v}{10\sqrt{2}}\right)^2 + \frac{v}{\sqrt{2}}.\frac{3v}{10\sqrt{2}} + 120 \quad \text{from (ii)}$$

$$\therefore \quad -5.\frac{9v^2}{200} + \frac{3v^2}{20} + 120 = 0$$

$$\text{or} \quad \frac{3v^2}{40} = 120 \quad \therefore \; v^2 = 40 \times 40$$

$$\text{ie} \quad v = 40$$

(iv) $\dot{y} = 0 \Rightarrow t = \frac{v}{10\sqrt{2}} = \frac{4}{\sqrt{2}}$

$$\therefore \max y = -5.\frac{16}{2} + \frac{40}{\sqrt{2}}.\frac{4}{\sqrt{2}} + 120 = 160$$

(b) (i) $E(n) = \left(1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + x^n\right) - 1 - nx$

$= \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + x^n$ since $\binom{n}{1} = n$

$= x^2\left(\binom{n}{2} + \binom{n}{3}x + \cdots + x^{n-2}\right)$

is divisible by x^2 (since $\binom{n}{2} + \cdots + x^{n-2}$ is an integer)

(ii) $E(2) = (1+x)^2 - 1 - 2x$

$= 1 + 2x + x^2 - 1 - 2x = x^2$ is div. by x^2

$\therefore$ Assume $(1+x)^n - 1 - nx = x^2 q$, q an integer, for $n \geq 2$

Then $E(n+1) = (1+x)^{n+1} - 1 - (n+1)x$

$= (1+x)(1+x)^n - 1 - (n+1)x$

$= (1+x)\left[x^2 q + 1 + nx\right] - 1 - nx - x$, using the assumption

$= q(1+x)x^2 + 1 + nx + x + nx^2 - 1 - nx - x$

$= q(1+x)x^2 + nx^2$

$= x^2(q(1+x) + n)$

is div by x^2 (since $q(1+x) + n$ is an integer)

$\therefore$ by induction $E(n)$ is div by x^2 for $n \geq 2$

Question 7

(a) (i) $\because f(x)$ is increasing for $x \geq 2$

(ii) $f(x) : x \geq 2, y \geq 0$

$\therefore f^{-1}(x) : x \geq 0, y \geq 2$

(iii) They must meet at $y = x$

$$\therefore (x-2)^2 = x$$

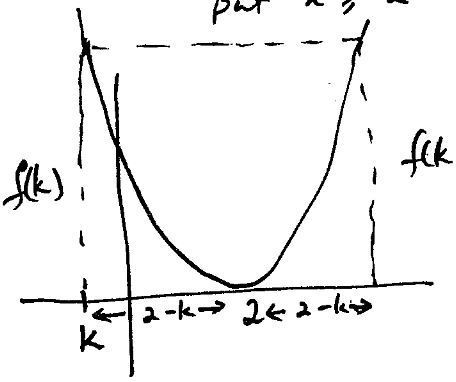
$$x^2 - 5x + 4 = 0$$

$$(x-1)(x-4) = 0$$

$$x = 1, 4$$

But $x \geq 2$ $\therefore$ They meet at $(4, 4)$

(iv)



$$\therefore f^{-1}(f(k)) = 2 + 2 - k = 4 - k$$

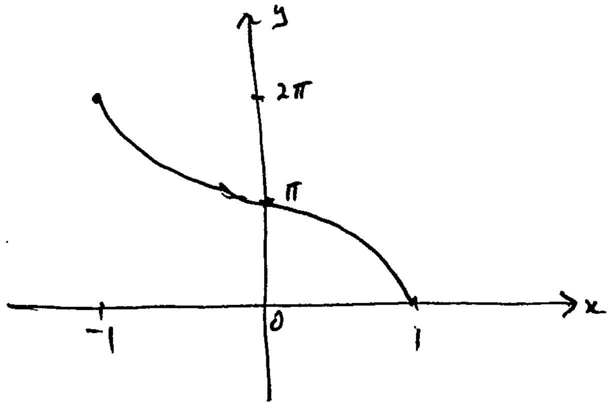
[ALTERNATIVES ABOUND]

e.g. $f(x) = (x-2)^2 = ((4-x)-2)^2$

$\Rightarrow$ if $k < 2$ $f^{-1}(f(k)) = f^{-1}(f(4-k))$ where $4-k > 2$

$= 4-k$

(b) (i)



(ii) $g'(x) = \dfrac{1}{\sqrt{1-(2x^2-1)^2}} \cdot 4x$

$= \dfrac{4x}{\sqrt{1-(4x^4-4x^2+1)}} = \dfrac{4x}{\sqrt{4x^2-4x^4}}$

$= \dfrac{4x}{2\sqrt{x^2}\sqrt{1-x^2}}$

$= \dfrac{2x}{(-x)\sqrt{1-x^2}}$ since $-1<x<0$

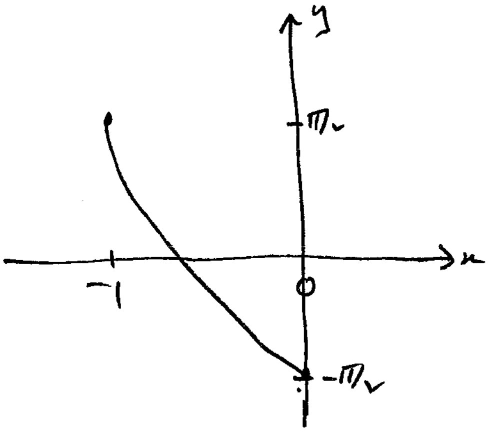
$= \dfrac{-2}{\sqrt{1-x^2}} = f'(x)$

(iii) From (ii), $\sin^{-1}(2x^2-1) = 2\cos^{-1}x + c$, c a constant

Put $x=0$, $-\frac{\pi}{2} = \pi + c$, $c = -\frac{3\pi}{2}$

i.e. $\sin^{-1}(2x^2-1) = 2\cos^{-1}x - \frac{3\pi}{2}$, $-1 \le x \le 0$

(iv) Using (i) and (iii),



MATHEMATICS EXTENSION 1 MARKING SCHEME 2009

Qn 1

(a) Ignore $+c$

(b) Allow 1 for $\tan\alpha = \dfrac{9-\frac{4}{5}}{1+9.\frac{4}{5}}$

(c) Allow 1 for $\left[\ln(e^x+1)\right]_0^1$

Allow 2 for decimal approxn $[\cdot 62 \ldots]$

(d) Allow 2 for
$$\begin{array}{r|l} & x \quad +1 \\ x^2-12 & x^3 + x^2 + x \\ & \quad \frac{-12x}{13x} + \frac{-12}{12} = R(x) \end{array}$$

(e) Allow 1 for $2x-1>0$ and $x+1>0$

Qn 2

(a) (i) Allow 1 for $\dfrac{dx}{d\theta} = \cos\theta$ & $\underline{\underline{\text{limits}}}$

Allow 1 for $J = \displaystyle\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta$

(ii) Allow 1 for $J = \frac{1}{2}\displaystyle\int_0^{\frac{\pi}{6}} 1-\cos 2\theta \, d\theta$

1 for $J = \frac{1}{2}\left[\theta - \dfrac{\sin 2\theta}{2}\right]_0^{\pi/6}$

(b) (i) Allow 1 for $1+\alpha = 0$

(c) (ii) Allow 1 for $\dfrac{1+t^2}{2t} + \dfrac{1-t^2}{2t}$

(Alternatives exist, of course)

Qn 3

(a) Allow 1 for $\frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right)$

IGNORE $+c$

(b) Allow 1 for $f'(x) = \cdot 4 + 2xe^{-x^2}$

Allow 1 for $x_1 = 1 - \dfrac{0\cdot 4 - e^{-1}}{0\cdot 4 + 2e^{-1}}$ or equivalent

Allow $\underline{\underline{3}}$ for $x_1 = \cdot 9717 \ldots$

(c) Allow 1 for $\dfrac{dv}{dt} = \dfrac{dv}{dx}\cdot\dfrac{dx}{dt} = v\dfrac{dv}{dx}$

(d) (i) Allow 1 for $\frac{1}{2}v^2 = -n^2\frac{x^2}{2} + c$ or equivalent

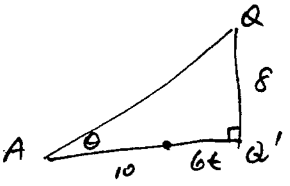
ie. $\frac{1}{2}v^2 = \left[-n^2\frac{x^2}{2}\right]_A^x$

(ii) **No** marks for $4 = n^2(A^2-36)$ and $16 = n^2(A^2-9)$

Allow 1 for $\frac{A^2-9}{A^2-36} = 4$ or equivalent

Allow 2 for $A = \sqrt{45}$ and $n = \frac{2}{3}$

Qu 4 (b) (i) Allow 1 for



ie. $AQ' = 10 + 6t$

(ii) Allow 1 for $\frac{d\tan\theta}{d\theta}\cdot\frac{d\theta}{dt} = \frac{d}{dt}\left(\frac{4}{5+3t}\right)$ or clear indication of same

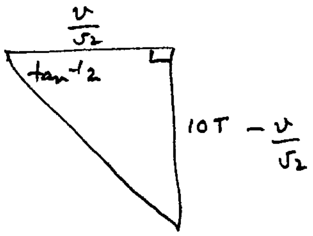
(iii) Allow 2 for $-\frac{27}{\pi}$ °/min or decimal approx^n 8.594...

Qu 5 (a) (ii) Allow 1 for $u_{k+1} = \binom{20}{k}(x^2)^{20-k}\left(\frac{b}{x}\right)^k$

Allow 1 for $40-3k=7$ and $40-3k=4$

(d) (v) Allow 1 for reason why grad $TR = \frac{-2}{p+q}$

Qu 6 (a) (ii) Allow 1 for



(iii) Allow 1 for $0 = -5\left(\frac{3v}{10\sqrt{2}}\right)^2 + \frac{v}{\sqrt{2}}\frac{3v}{10\sqrt{2}} + 120$ or equivalent

ie $-5T^2 + T.\frac{10T}{3} + 120 = 0$

(iv) Allow 1 for $-10t + \frac{v}{\sqrt{2}} = 0$

IGNORE UNITS

(d) (ii) Allow 1 for E(2)

Allow 1 for clear assumption

IGNORE CONCLUSION

Qn 7 (a) (iv) No marks for $f^{-1}f(k) = k$

LOTS OF ALTERNATIVES, OF COURSE

e.g. $f^{-1}(x)$: $x = (y-2)^2$, $x \geq 0$, $y \geq 2$

$\Rightarrow y = 2 + \sqrt{x}$ [DO NOT ALLOW 1 for this]

$\therefore f^{-1}f(k) = f^{-1}((k-2)^2) = 2 + \sqrt{(k-2)^2}$

$= 2 + 2 - k$ since $k - 2 < 0$

$= 4 - k$

(b) (ii) Allow 1 for $g'(x) = \dfrac{1}{\sqrt{1-(2x^2-1)^2}} \cdot 4x$

1 for $g'(x) = \dfrac{4x}{2\sqrt{x^2}\sqrt{1-x^2}}$

(iii) Allow 1 for $\sin^{-1}(2x^2-1) = 2\cos^{-1}x + c$